

Cosmological Growth History of Supermassive Black Holes and Demographics in the High- z Universe : Do Lyman-Break Galaxies Have Supermassive Black Holes?

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ABSTRACT

We study the demographics of supermassive black holes (SMBHs) in the local and high- z universe. We use the continuity equation of the population of SMBHs as the leading principle. We consider three physical processes for the growth of SMBHs: mass accretion, mergers, and direct formation of SMBHs. The mass accretion history of SMBHs is estimated by the hard X-ray luminosity functions (HXLFs) of AGNs. First, we compare the mass accretion history at $z > 0$ with optical luminosity functions (OLFs) of QSOs previously studied and that with HXLFs. We conclude that the constraints on parameters of mass accretion (energy conversion efficiency, ϵ and Eddington ratio, f_{Edd}) based on the continuity equation appear to be adequate using HXLFs rather than OLFs. The sub-Eddington case ($f_{\text{Edd}} < 1$) is allowed only when we use HXLFs. Next, we extend the formulation and we can obtain the upper limit of the cumulative mass density of SMBHs at any redshifts. For an application, we examine if Lyman-Break galaxies (LBGs) at $z \sim 3$ already have SMBHs in their centers which is suggested by recent observations. We tentatively assume the presence of SMBHs in LBGs and that their mass, M_{BH} is proportional to the stellar mass of LBGs, M_* with the mass ratio $\xi = M_{\text{BH}}/M_*$. If most of LBGs already has massive SMBHs at $z \sim 3$, the resultant mass density of SMBHs at $z \sim 0$ should exceed the observational estimate because such SMBHs should further grow by accretion. Therefore, we can set the upper limit of the probability that one LBG has a SMBHs. Since the merger rates and direct formation rates of SMBHs are uncertain, we consider two limiting cases : (i) mergers and/or direct formations are not negligible compared with mass accretion and (ii) mass accretion is the dominant process to grow the SMBHs. The special conditions should be met in order that a large part of LBGs have SMBHs in both cases. In case (i), we may

assume the constant parameters of mass accretion of AGNs for simplicity. Then, large energy conversion efficiency and frequent mergers and/or direct formations at $z > 3$ are needed so that a large part of LBGs have SMBHs with $\xi = 0.002$ to 0.005 . Whereas, in case (ii), energy conversion efficiency should be mass dependent and the constraint is strict; the fraction of LBGs which have SMBHs must be less than 10-40%. In both cases, the possibility that nearly all LBGs have SMBHs with large mass ratio, such as $\xi \geq 0.005$, is reliably ruled out.

Subject headings: black hole physics — galaxies: active — galaxies : evolution — galaxies: luminosity function, mass function — galaxies : nuclei — quasars: general

1. Introduction

Today, there is a wide consensus that almost all local galaxies harbor supermassive black holes (SMBHs) in their centers (e.g. Begelman 2003, Kormendy & Richstone 1995). We can estimate the mass of these SMBHs with dynamical motion of gases or stars around the center of galaxies (e.g. Ghez et al. 1998, Richstone et al. 1998, Gebhardt et al. 2003). It is well-known that there are some correlations between central SMBHs and its galactic bulges. For example, the mass of SMBHs is proportional to the mass of their galactic bulges; $M_{\text{BH}}/M_{\text{bulge}} \sim 0.001 - 0.006$, though its tightness and linearity is still under discussion (Kormendy & Richstone 1995, Maggorian et al. 1998, Laor 2001, McLure & Dunlop 2002, Marconi & Hunt 2003). Another tight correlation is so-called $M_{\text{BH}} - \sigma$ relation, where σ is the stellar velocity dispersion in the bulges. This relation is expressed as the power-law form, $M_{\text{BH}} \propto \sigma^{3.8-4.8}$ (e.g. Gebhardt et al. 2000, Ferrarese & Merritt 2000, Tremaine et al. 2002). Recently, Ferrarese (2002) and Baes et al. (2003) argue that SMBH mass is related to the dark halo mass beyond the bulge, $M_{\text{BH}} \propto M_{\text{DM}}^{1.27-1.82}$. Graham et al. (2001) point that there is another correlation that bulges with more massive SMBHs have steeper central cusps. In spite of discoveries of many correlations and great efforts which have been devoted to clarify the physical meanings of these correlations (e.g. Silk & Rees 1998, Ostriker 2000, Umemura 2001, Adams, Graff & Richstone 2001), we do not have a consensus yet regarding physical link between the formation process of SMBHs and that of their host galaxies (bulges).

To understand how the central SMBHs formed and how their formation history is related to that of galaxies, we must at least know when SMBHs were formed in galaxies. Therefore, it is very important to know whether high- z (not active) galaxies already had SMBHs or not. In this respect, it is interesting to note that the so-called Lyman-Break technique, for example, has made it possible to discover a significant number of high- z (> 3) galaxies, i.e.,

Lyman-Break Galaxies (LBGs, Steidel et al. 1996). These LBGs are widely known as high- z starburst galaxies and their typical starformation rate is $10^2 - 10^3 M_\odot \text{yr}^{-1}$. Papovich et al. (2001) and Shapley et al. (2001) applied the population synthesis models to infer UV-to-optical spectrum of LBGs at their rest frame and examined various properties of LBGs. Despite uncertainties their results are consistent at some points; LBGs are typically young (their ages are several hundred Myr since last star-formation events) and small (their typical stellar mass is $\sim 10^{10} M$), though there were some old LBGs.

Ridgway et al. (2001) point that the sizes and magnitudes of the host galaxies of the radio-quiet QSOs at $z \sim 2 - 3$ are similar to those of LBGs in HST imaging study. Steidel et al. (2002) observed about 1000 LBGs and argue that 3% of these LBGs are optically faint AGNs based on their UV spectrum. Nandra et al. (2002) confirmed by using *Chandra* observatory that the same number fraction, 3%, of LBGs are extremely bright in the rest-frame hard X-ray band, and asserted that these LBGs show AGN activity. They discussed that this number fraction, 3% may reflect the duty cycle of mass accretion to SMBHs in LBGs. That is, if the typical age of LBGs is 300 Myr (e.g. Shapley et al. 2001) and the accretion timescale is 10^7 yr, remaining 97% of LBGs may have inactive SMBHs. This situation may be compared with the local universe. In the local universe, almost all galaxies harbor SMBHs in their centers, but there are much inactive galaxies than AGNs. The pioneering work (Dressler, Thompson & Shectman 1985) found that the number fraction of AGNs to all galaxies is only several %, though this fraction recently increases to 20-40 % due to the accurate observations (Kauffmann et al. 2003, Miller et al. 2003). Furthermore, Vestergaard (2003) and Vestergaard (2004) discusses that the LBGs with AGN activity may have SMBHs of order $10^8 M_\odot$ from the virial mass estimate using C IV line width. If these LBGs have typical stellar mass of $M_* \sim 10^{10} M_\odot$, the mass ratio becomes $M_{\text{BH}}/M_* \sim 0.01$, which is larger than $M_{\text{BH}}/M_{\text{bulge}} \sim 0.001 - 0.006$ of local galaxies. Incidentally, normal galaxies and AGNs have the same mass ratio, $M_{\text{BH}}/M_{\text{bulge}}$, in the local universe (Wandel 2002). Whereas, Granato et al. (2001) found that LBGs may be equivalent to the galaxies which are in the pre-AGN phase based on their semi-analytic model of the joint formation of QSOs and galaxies. Kawakatu et al. (2003) argue that it may be physically difficult to feed a black hole in optically thin galaxies, such as LBGs, based on the idea by Umemura (2001). In the local universe, almost all the SMBHs observationally accompany the stellar spheroidal (bulge) component (M33, which is the bulgeless galaxy, appears to have no SMBHs (Gebhardt et al. 2001)), therefore, the bulge component may be indispensable to form a SMBH. If LBGs are still in the initial phase of forming the bulge component (Matteucci & Pipino 2002), LBGs may not have SMBHs.

Of course, it is difficult to resolve the central parsec scale region of LBGs and search the (inactive) SMBHs observationally. In this paper, we theoretically examine the possi-

bility that LBGs have SMBHs from the viewpoints of the demographics. We consider the cosmological accretion history derived from the observed hard X-ray (2-10keV) luminosity functions (HXLFs) of AGNs at $z > 0$ (Ueda et al. 2003, hereafter UAOM03). Yu & Tremaine (2002) (hereafter YT02) use the continuity equation for the number density of SMBHs (see also Soltan 1982, Small & Blandford 1992) and show that the accreted mass estimated with optical LFs (OLFs) of QSOs at $z > 0$ is comparable with the SMBH mass density of the local universe. However, we can see only a part of various populations of AGNs in the optical band, thus the analysis with hard X-ray LFs, which include absorbed AGNs, is important. We consider the cosmological accretion history estimated with HXLFs and compare it with the accretion history estimated with OLFs (YT02). Furthermore, we basically extend the formulation of YT02 and crudely estimate how many and how massive SMBHs are allowed to exist in the high- z universe. If most LBGs already had very massive SMBHs at $z \sim 3$, the accreted mass into SMBHs until $z \sim 0$ should exceed the BH mass density of the local universe. In this way, we will be able to set constraints on the mass density of SMBHs in LBGs.

Below, we use the continuity equation of SMBH population as the leading principle. In §2, we first transform the continuity equation to the useful integrated form (§2.1). We then compare the cosmological accretion history at $z > 0$ with the local population of SMBHs and constrain the physical parameters; the energy conversion efficiency, ϵ and the Eddington ratio, $f_{\text{LBG}} (\equiv L_{\text{bol}}/L_{\text{Edd}})$ (§2.2). In this time, we compare the accretion history with HXLFs and that with OLFs. In §3, we extend YT02’s formulation and derive the upper limit of the population of SMBHs at $z \sim 3$ (§3.1). Furthermore, we consider the tentative SMBHs in LBGs with mass ratio, $\xi \equiv M_{\text{BH}}/M_*$ (M_* is the stellar mass of LBGs), and crudely estimate the upper limit of the probability that one LBG has a SMBH (§3.2). In §4, we summarize and discuss the results. Throughout this paper, we adopt the Λ -dominant universe and the cosmological parameters are $(\Omega_M, \Omega_\Lambda, h) = (0.3, 0.7, 0.7)$.

2. Cosmological Accretion History v.s. Local Population of Supermassive Black Holes

2.1. Continuity Equation of the Population of SMBHs

We begin with the continuity equation for the number density of SMBHs: $n(M, t) \equiv dN/dM(M, t)$; that is

$$\frac{\partial n}{\partial t} + \frac{\partial[n \cdot \langle \dot{M} \rangle]}{\partial M} = \gamma_{\text{merge}}(M, t) + \gamma_{\text{form}}(M, t) \quad (1)$$

(Soltan 1982, Small & Blandford 1992, YT02). Hereafter, we simply express the black hole mass as M . The source terms on the right-hand side, $\gamma_{\text{merge}}(M, t)$ and $\gamma_{\text{form}}(M, t)$, represent the SMBH-SMBH merger rates (e.g. Haehnelt 1994) and the rates of the direct formation of SMBHs by, say, collapse of the supermassive stars (SMS) ($\sim 10^5 M_\odot$, or may be more massive). We can express these terms explicitly. The merger term is,

$$\gamma_{\text{merge}}(M, t) = \frac{1}{2} \int_0^\infty dM_1 \int_0^\infty dM_2 \Omega(M_1, M_2) [\delta(M - M_1 - M_2) - \delta(M - M_1) - \delta(M - M_2)], \quad (2)$$

where $\Omega(M_1, M_2)$ is the distribution function which represents the merger rates among the SMBHs of mass M_1 and M_2 . The factor $1/2$ is necessary to correct the double counts of the merger rates in the integral in eq.(2). Several recent studies investigate the distribution function of mergers, $\Omega(M_1, M_2)$ with the semi-analytic model of galaxies and QSOs (e.g. Kauffmann & Haehnelt 2000, Volonteri, Haardt & Madau 2003). However, the key problem of the SMBH merger is the timescale. That is, the timescale to refill the "loss cone" of the SMBH binary (see Begelman et al. 1980 or Milosavijević & Merritt 2003). Although the hardening timescale of the binary have been actively studied, this problem is still under discussion (Makino 1997, Milosavijević & Merritt 2001, Milosavijević & Merritt 2003, Makino & Funato 2004). Therefore, we treat $\Omega(M_1, M_2)$ as the unknown function. Here, we assume that the total mass of SMBHs is conserved through merger events. As for γ_{form} , we only note that this numerical value is not negative,

$$\gamma_{\text{form}}(M, t) \geq 0. \quad (3)$$

Recently, the gravitational collapse of SMS is being studied with the general relativistic numerical simulations (e.g. Shapiro & Shibata 2002, Shibata & Shapiro 2002). However, it is still uncertain how and when the initial condition of the simulations is realized. Eventually, these source terms are highly uncertain and we do not know whether these terms exist or not (the merger term will be at least non-negligible (e.g. Merritt & Ekers 2003)). YT02 successfully avoid this difficulty by multiplying eq.(1) by the function,

$$f(M, M') = \begin{cases} 0, & \text{for } M' < M \\ M' - M, & \text{for } M' > M \end{cases} \quad (4)$$

and integrate by M' from 0 to ∞ and by t from 0 to t_0 (cosmological time at $z = 0$), assuming that $n(M, t) \rightarrow 0$ in the limit of $t \rightarrow 0$ and $M' \rightarrow \infty$. The result is

$$G_{\text{local}}(M, t_0) = G_{\text{acc}}(M, t_0) + G_{\text{merge+form}}(M, t_0). \quad (5)$$

Here, $G_{\text{local}}(M, t_0)$, $G_{\text{acc}}(M, t_0)$ and $G_{\text{merge+form}}(M, t_0)$ are defined by

$$G_{\text{local}}(M, t_0) \equiv \int_M^\infty (M' - M) n(M', t_0) dM', \quad (6)$$

$$G_{\text{acc}}(M, t_0) \equiv \int_M^\infty dM' \int_0^{t_0} n(M', t) \langle \dot{M}' \rangle dt, \quad (7)$$

$$G_{\text{merge+form}}(M, t_0) \equiv \int_M^\infty \int_0^{t_0} (M' - M) (\gamma_{\text{merge}}(M', t) + \gamma_{\text{form}}(M', t)) dM' dt, \quad (8)$$

where $\langle \dot{M}' \rangle$ is the average mass accretion rate into the SMBHs of mass M' . Since $G_{\text{merge+form}}(M, t_0)$ is 0 or positive, we obtain the inequality,

$$G_{\text{local}}(M, t_0) \geq G_{\text{acc}}(M, t_0). \quad (9)$$

These mathematical derivation of eq.(9) is the same as that in YT02, whereas we explain the physical meaning of eq.(9) here. To do so, we transform inequality (9) as

$$\int_M^\infty M' n(M', t_0) dM' \geq \int_M^\infty dM' \int_0^{t_0} n(M', t) \langle \dot{M}' \rangle dt + M \int_M^\infty n(M', t_0) dM'. \quad (10)$$

In this form, we can understand the meanings of each term. Clearly, the left-hand side of eq.(10) represents the mass density contained by SMBHs more massive than M at $z = 0$. The first term of the right-hand side represents the mass accreted into the SMBHs more massive than M per unit volume during $0 \leq t \leq t_0$ (i.e. $z \geq 0$). The second term on the right-hand side represents another contribution; that is, the mass density of SMBHs at the moment that SMBHs become more massive than M . In our consideration, the SMBHs more massive than M can form through the mass accretion, mergers, or gravitational collapse of the SMS. The mass of a SMBH continuously grows by the mass accretion, whereas the mass of a SMBH grows discontinuously by the mergers and/or direct formation. Therefore, the mass of SMBHs at the moment that SMBHs become more massive than M is equal to M for the case with the mass accretion, whereas it is more than M otherwise. The total number of SMBHs which became more massive than M until $z = 0$ is necessarily larger than $\int_M^\infty n(M', t_0) dM'$ since the mergers reduce the number of SMBHs. Totally, the lower limit of the contribution of the mass density of SMBHs at the moment that SMBHs become more massive than M is $M \int_M^\infty n(M', t_0) dM'$, which is the second term on the right-hand side of eq.(10). The physical meaning of this term may be considered as the “advection” of SMBH mass across the mass M in terms of the continuity equation of the population of SMBHs. This “advection” term is the heart of this inequality.

We can calculate $G_{\text{local}}(M, t_0)$ with the local mass function of SMBHs. We adopt the mass function of SMBHs given by Aller & Richstone (2002) (hereafter AR02) based on the luminosity functions of galaxies at $z = 0$ for each morphology. They convert the local LF of normal galaxies to the mass function of SMBHs with some empirical relations among total luminosity, bulge luminosity, velocity dispersion and SMBH mass. One of these relations

is $M - \sigma$ relation mentioned above. AR02 derive the two local mass functions of SMBHs for different galactic LFs ; mass function (a) is derived from LFs by Marzke et al. (1994), whereas mass function (b) is derived from LFs by Madgwick et al. (2002). Below, we show the results of the calculation for the cases with mass function (a) and refer the results of the other cases in text. We use the fitting formulae for the mass functions of SMBHs given by AR02. With the mass functions (a) and (b), the local mass density of SMBHs more massive than $10^6 M_\odot$ is $(4.8 \pm 1.6)h^2 \times 10^5 M_\odot \text{Mpc}^{-3}$ and $(6.9 \pm 1.4)h^2 \times 10^5 M_\odot \text{Mpc}^{-3}$ respectively. YT02 adopt the LFs based on the SDSS (Sloan Digital Sky Survey) and their calculated value is $(2.5 \pm 0.4) \times 10^5 M_\odot \text{Mpc}^{-3}$ with $h = 0.65$. Salucci et al. (1999) and Merritt & Ferrarese (2001) also obtain similar results.

To calculate $G_{\text{acc}}(M, t_0)$, there is a difficulty in that we do not know the number density of SMBHs at cosmological time t , $n(M, t)$, and the average accretion rate of SMBHs, $\langle \dot{M} \rangle$. Here, we assume that the mass accretion into SMBHs occurs mainly at bright AGN phase. The average accretion rate of AGNs is assumed to be

$$\langle \dot{M} \rangle = \frac{(1 - \epsilon)L_{\text{bol}}}{\epsilon c^2}, \quad (11)$$

where L_{bol} is the bolometric luminosity of AGNs and ϵ is the energy conversion efficiency. With eq.(11), we can express $G_{\text{acc}}(M, t_0)$ as

$$G_{\text{acc}}(M, t_0) = \int_{L(M)}^{\infty} dL' \int_0^{t_0} \frac{d\Phi(L', t)}{dL'} \frac{(1 - \epsilon)L_{\text{bol}}}{\epsilon c^2} dt. \quad (12)$$

Here, $d\Phi(L, t)/dL$ is the LFs of AGNs and we use observed HXLFs (UAOM03). Very recently, UAOM03 build the HXLFs of AGNs over the wide luminosity range, $\log(L_x(2 - 10\text{keV}) \text{ erg/s}) = 41.5 - 46.5$, and wide redshift range, $z = 0 - 3$, from the combination of surveys with *HEAO1*, *ASCA* and *Chandra* satellites. The shape of HXLFs of AGNs at $z = 0$ are fitted by so-called double power-law function well. That is,

$$\frac{d\Phi(L_x, z = 0)}{d \log L_x} = A [(L_x/L_*)^{\gamma_1} + (L_x/L_*)^{\gamma_2}]^{-1}. \quad (13)$$

In the above expression, γ_1 and γ_2 represent the inclinations of LFs at the lower and higher luminosity ends respectively, and L_* represent the luminosity of the “knee” of LFs. The redshift evolution of HXLFs are described as the luminosity-dependent density evolution (LDDE), which is suggested by Miyaji et al. (2001) for the evolution of the soft X-ray luminosity functions of AGNs. The LDDE model is expressed as

$$\frac{d\Phi(L_x, z)}{d \log L_x} = \frac{d\Phi(L_x, 0)}{d \log L_x} \times e(z, L_x), \quad (14)$$

where

$$e(z, L_x) = \begin{cases} (1+z)^{p1}, & \text{for } z < z_c(L_X) \\ e(z_c)[(1+z)/(1+z_c(L_X))]^{p2}, & \text{for } z \geq z_c(L_X), \end{cases} \quad (15)$$

and

$$z_c(L_X) = \begin{cases} z_c^*, & \text{for } L_X \geq L_a \\ z_c^*(L_X/L_a)^\alpha, & \text{for } L_X < L_a. \end{cases} \quad (16)$$

The parameters which represent redshift dependence of $e(z, L_X)$ are $p1 > 0$ and $p2 < 0$. Then, $z_c(L_X)$ corresponds to the peak redshift at which the number density of AGNs with luminosity of L_X is the maximum. The cut-off redshift, $z_c(L_X)$, increases as the luminosity increases ($\alpha > 0$) (see Fig.10 of UAOM03). That is, the number density of the more luminous AGNs reaches the maximum at the higher redshift and decrease as $z \rightarrow 0$ (see Fig.12 of UAOM03). We simply extrapolate the fitting formulae given by above LDDE model at $z > 3.0$.

To calculate $G_{\text{acc}}(M, t_0)$, we need the relation between the hard X-ray luminosity, L_X and the bolometric luminosity, L_{bol} . We assume the relation as $L_{\text{bol}} = C_X L_X$, where C_X is constant value $C_X = 25.0$ (Alonso-Herrero et al. 2002). Generally, the ratio of B-band (4400 Å), hard X-ray (2-10 keV) luminosity to the bolometric luminosity, $C_B = L_B/L_{\text{bol}}$ and C_X should depend on the mass accretion rate and the black hole mass. This may be important to compare the local mass function of SMBHs with the mass accretion history estimated with OLFs and that estimated with HXLFs. We will discuss this issue in the next subsection in some details. We suppose that the QSOs radiate as $L_{\text{bol}} = f_{\text{Edd}} L_{\text{Edd}} = 1.5 \times 10^{38} f_{\text{Edd}} (M/M_\odot) (\text{erg/s})$. The parameters here are the energy conversion efficiency, ϵ , and the Eddington ratio, f_{Edd} .

2.2. Comparison between the Mass Accretion History with HXLFs and that with OLFs

Here, we compare the mass accretion history estimated with HXLFs and that with OLFs based on the continuity equation. For both cosmological accretion history with HXLFs and that with OLFs, the relation to the local mass function of SMBHs can be investigated with ineq.(9) derived from the continuity equation.

YT02 noted that their calculated $G_{\text{acc}}(M, t_0)$ with OLFs of QSOs with the typical parameter, $\epsilon = 0.1$ and $f_{\text{Edd}} = 1.0$ is larger than $G_{\text{local}}(M, t_0)$, especially in the large mass range, $M > 10^7 M_\odot$ (inequality (9) is not satisfied). That is, there is the problem of the overaccretion compared to the local mass function of SMBHs. YT02 discussed the possible origin of this discrepancy. For example, they argue that the optically bright QSOs may

radiate at large mass-to-energy conversion efficiency, ϵ (e.g. $\epsilon \geq 0.2$). However, we can see only a part of the light emitted in the mass accretion process to SMBHs in the optical band. This is partly because the optical luminosity emitted through the mass accretion can be absorbed by obscuring torus (Type II AGNs) (e.g. Antonucci 1993). We can miss the reddened QSOs due to the dust extinction in the host galaxies when the typical color selection criteria (e.g. UV excess) is adopted to build the QSO LFs (e.g. Webster et al. 1995). Thus, we calculate $G_{\text{acc}}(M, t_0)$ with HXLFs and compare it to $G_{\text{acc}}(M, t_0)$ with OLFs and to $G_{\text{local}}(M, t_0)$. Of course, as YT02 noted, it seems to make the problem worse to use the HXLFs. This is because HXLFs include extra populations of AGNs besides the optically bright AGNs. However, this is not the case. The redshift evolution of HXLFs and that of OLFs are different, then we can make the situation better (see below).

The upper panel of Fig.1 represents the results with the local mass function of SMBHs (a). For OLFs of QSOs, we adopt ones given by Boyle et al.(2000) at $0.35 < z < 2.3$ derived from about 6,000 QSOs detected by 2QZ survey and simply extrapolate it at $z > 2.3$. We assume the constant bolometric correction, $C_B = 11.8$ (Elvis et al. 1994). YT02 use the same OLFs and adopt the same assumptions. Like YT02's result, our results show that the inequality (9) is not satisfied in all mass range (especially at high mass end) with OLFs and typical parameters, $(\epsilon, f_{\text{Edd}}) = (0.1, 1.0)$. If we use mass function (b), the calculated $G_{\text{local}}(M, t_0)$ is larger than (a) by a factor 1.4, but equation (9) is not still satisfied at $M_{\text{BH}} \geq 10^{7.5} M_{\odot}$.

However, Fig.1 also shows that calculated $G_{\text{acc}}(M, t_0)$ with HXLFs with the same parameter set do not exceed $G_{\text{local}}(M, t_0)$ in $M < 10^{8.5} M_{\odot}$. At first sight, this looks strange. Since in HXLFs optically absorbed AGNs and reddened AGNs are considered besides the optically bright QSOs, $G_{\text{acc}}(M, t_0)$ calculated based on HXLFs should become larger than $G_{\text{acc}}(M, t_0)$ based on OLFs. The origin of this puzzle lies in our assumption on C_B and C_X . We assume that both of C_B and C_X are constant; $C_B = 11.8$ and $C_X = 25.0$. If this were actually the case, the ratio between B-band luminosity and hard X-ray luminosity, L_B/L_X , must be constant. However, UAOM03 compare their HXLFs of only optical type-I AGNs to OLFs by Boyle et al. (2000) and find that L_X is not proportional to L_B . They obtained a power-law relation between the 2keV and 2500Å luminosities, $l_X \propto l_O^{0.7}$ in the $(\Omega_M, \Omega_{\Lambda}, h) = (1.0, 0.0, 0.5)$ universe. This relation is approximately transformed to the relation between $L_X(2 - 10\text{keV})$ and M_B as

$$M_B = -\frac{7}{2} \log[L_X(h/0.5)^{-2} \text{ (ergs/s)}] + 131.5. \quad (17)$$

That is, $L_B \propto L_X^{1.4}$. Therefore, the assumption that both of C_B and C_X are constant is not justified. As noted by UAOM03, if L_B/L_X were constant, the redshift evolution of OLFs would be more quick than that of HXLFs. The bright end of OLFs grows more quickly than

HXLFs as z increases. Then, calculated $G_{\text{acc}}(M, t_0)$ based on OLFs is larger than that based on HXLFs. Kawaguchi, Shimura & Minehige (2001) build a disk-corona model for the broad-band spectral energy distributions (SEDs) of AGNs. Based on their model, Hosokawa et al. (2001) and Alonso-Herrero et al. (2002) calculated how C_B and C_X depend on the black hole mass, M and on the mass accretion rate, \dot{M} . Though their dependence on M and \dot{M} is not simple, C_X comparatively remains constant for varying M and \dot{M} . Therefore, we adopt constant $C_X = 25.0$ as noted above and use this value for the calculations below. Incidentally, if L_X is proportional to the bolometric luminosity, L_{bol} , eq.(17) may be expressed as $C_B \propto L_{\text{bol}}^{-0.4} \propto M^{-0.4}$ with constant Eddington ratio. Alonso-Herrero et al. (2002) show the similar dependence, $C_B \propto M^{-0.25}$ based on Kawaguchi, Shimura & Mineshige (2001), though the dependence is somewhat weaker.

Even if we use HXLFs, the calculated $G_{\text{acc}}(M, t_0)$ with $(\epsilon, f_{\text{Edd}}) = (0.1, 1.0)$ becomes larger than $G_{\text{local}}(M, t_0)$ in the high mass end ($M > 10^{8.5} M_{\odot}$). However, this may be an inevitable result. We calculate $G_{\text{local}}(M, t_0)$ with the local mass function of SMBHs derived with the Schechter-type LFs of galaxies, which declines exponentially at the bright end, whereas we calculate $G_{\text{acc}}(M, t_0)$ with the HXLFs which is power-law function at the bright end. YT02 point that the QSO LFs at the bright end or $M - \sigma$ relation at the high mass end may be uncertain. Then, we use the energy conversion efficiency, ϵ , and Eddington ratio, f_{LBG} as constant free parameters to calculate $G_{\text{acc}}(M, t_0)$, and search the region of the parameter space of $(\epsilon, f_{\text{LBG}})$ where inequality eq.(9) is satisfied only at $M \leq M_0$ for given M_0 . We choose $M_0 = 10^{8.0} M_{\odot}$ and $M_0 = 10^{8.5} M_{\odot}$. Of course, the allowed parameter space becomes narrower as M_0 increases, since the discrepancy we show in Fig.1 is more prominent at the high mass end.

Fig.2 represents the results for the cases with the local mass function of SMBHs (a). The allowed region for each M_0 is above the labeled lines respectively. We set the upper limit of ϵ as $\epsilon = 0.3$, which is the maximum value for the thin-disk accretion models (Thorne 1974). If we use mass function (b), the allowed region for each M_0 becomes a little wider than (a). This is consistent with that the calculated $G_{\text{local}}(M, t_0)$ with (b) is larger and it is easier to satisfy the inequality (9) than the cases with mass function (a). As shown in Fig.2, to satisfy ineq. (9) at $M \leq 10^{8.5} M_{\odot}$, the allowed region is not so large if we demand $f_{\text{Edd}} \leq 1$. If $f_{\text{Edd}} \sim 1$, ϵ can be 0.1-0.3. However, only the large energy conversion efficiency ($\epsilon \sim 0.3$) is possible as f_{Edd} decreases ($f_{\text{Edd}} \sim 0.3$). This conclusion will not change, even if we adopt mass function (b). In Fig.2, we show the same parameter regions when we use OLFs of QSOs to calculate $G_{\text{acc}}(M, t_0)$. To satisfy inequality (9) to high mass end, high Eddington ratio ($f_{\text{Edd}} > 1$) and high energy conversion efficiency ($\epsilon > 1$) are needed. As shown above, the cosmological accretion history estimated with HXLFs naturally satisfy ineq. (9) to high mass end with typical parameters (especially $f_{\text{Edd}} < 1$) and constant luminosity ratio, $C_X = 25.0$.

Therefore, we conclude that the accretion history with HXLFs is more plausible than that with OLFs. Below, we continue to use HXLFs to estimate the cosmological accretion history of SMBHs.

In the lower panel of Fig.1, we plot the calculated $G_{\text{acc}}(M, t_0)$ at points 2 and 3 in Fig.2. As shown in Fig.1, if we adopt constant parameters, the quantity, $G_{\text{acc}}(M, t_0) - G_{\text{local}}(M, t_0)$, increases as M_{BH} decreases to satisfy ineq. (9) upto high-mass end (large M_0). From eq.(5), this means that the contribution of mergers and/or direct formation of SMBHs becomes important at the low mass end. If more massive halos possess more massive SMBHs, this may be the case since mergers between small halos occur more frequently compared with mergers between large halos under the CDM cosmology.

In the next section, we consider the possibility that LBGs have inactive SMBHs as in the case of the normal galaxies at $z = 0$. To do so, we calculate the accreted mass into SMBHs during $0 < z < 3$. In this case, we should choose parameter set of $(\epsilon, f_{\text{LBG}})$ so that ineq.(9) is satisfied. Therefore, we use $(\epsilon, f_{\text{LBG}})$ indicated by points in Fig.2. As discussed above, with constant parameters, contribution of the mergers and/or direct formation of SMBHs to grow SMBHs becomes important in the low mass end. Below, we adopt the constant parameters for the case of non-negligible contribution of mergers and/or direct formation of SMBHs compared with mass accretion.

3. Demographics of SMBHs in the High- z Universe

3.1. Upper Limit of the Cumulative Mass Density of SMBHs at High- z Universe

In this section, we investigate the possibility that LBGs have SMBHs. Our approach is the statistical one. That is, if there were already too many or too massive SMBHs in the universe of $z \sim 3$, the resultant mass density of SMBHs at $z \sim 0$ should exceed the observed local mass density of SMBHs, that is a contradiction, because SMBH mass density can never decrease. The basic formulation is similar to that presented in the previous section. Here, we multiply the continuity equation, eq.(1), by $f(M, M')$ defined by eq.(4) and integrate it by M' from M to ∞ and by t from $t_{z=3}$ (cosmological time at $z = 3$) to t_0 . We obtain

$$\begin{aligned} & \int_M^\infty M' (n(M', t_0) - n(M', t_{z=3})) dM' - \int_M^\infty \int_{t_{z=3}}^{t_0} M \frac{\partial n}{\partial t} dM' dt \\ &= \int_M^\infty \int_{t_{z=3}}^{t_0} n(M', t) \langle \dot{M}' \rangle dM' dt \end{aligned}$$

$$+ \int_M^\infty \int_{t_z=3}^{t_0} (M' - M) (\gamma_{\text{merge}}(M', t) + \gamma_{\text{form}}(M', t)) dM' dt, \quad (18)$$

where $n(M, t_0)$ is the local mass function of SMBHs, $dN/dM(M, t_0)$, as we defined in the previous section. We substitute eq.(1) for the second term on the left-hand side of eq.(18), we obtain the equation,

$$F_{\text{local}}(M) - F_{z=3}(M) - \hat{F}_{\text{acc}}(M) = F_{\text{acc}}(M) + F_{\text{merge+form}}(M). \quad (19)$$

Each terms is defined as below,

$$F_{\text{local}}(M) \equiv \int_M^\infty M' n(M', t_0) dM', \quad (20)$$

$$F_{z=3}(M) \equiv \int_M^\infty M' n(M', t_{z=3}) dM', \quad (21)$$

$$\hat{F}_{\text{acc}}(M) \equiv \int_{t_z=3}^{t_0} M n(M, t) \langle \dot{M} \rangle dt, \quad (22)$$

$$F_{\text{acc}}(M) \equiv \int_M^\infty \int_{t_z=3}^{t_0} n(M', t) \langle \dot{M}' \rangle dM' dt, \quad (23)$$

$$F_{\text{merge+form}}(M) \equiv \int_M^\infty \int_{t_z=3}^{t_0} M' (\gamma_{\text{merge}}(M', t) + \gamma_{\text{form}}(M', t)) dM' dt. \quad (24)$$

We note that only SMBH-SMBH mergers, in which the mass of two SMBHs, M_1 and M_2 , satisfy the condition that $M_1 > M$ and $M_2 > M$, do not contribute to $F_{\text{merge+form}}(M)$. This is because these mergers do not change the mass density contained in SMBHs more massive than M . As discussed in the previous section, $F_{\text{merge+form}}(M)$ is not negative, thus we obtain the inequality,

$$F_{z=3}(M) \leq F_{\text{local}}(M) - F_{\text{acc}}(M) - \hat{F}_{\text{acc}}(M). \quad (25)$$

The physical meaning of each term is evident. The cumulative mass density of SMBHs at $z = 0$, $F_{\text{local}}(M)$, is calculated with the local mass function given by AR02. The accreted mass into SMBHs more massive than M during $0 < z < 3$, $F_{\text{acc}}(M)$, is calculated by

$$F_{\text{acc}}(M) = \int_{L(M)}^\infty dL' \int_{t(z=3)}^{t_0} \frac{d\Phi(L', t)}{dL'} \frac{(1 - \epsilon)L'_{\text{bol}}}{\epsilon c^2} dt, \quad (26)$$

which resembles $G_{\text{acc}}(M, t_0)$ defined by eq.(12). Furthermore, the mass “advection” of SMBHs across the boundary mass M (already mentioned in §2.1) by the mass accretion during $0 < z < 3$, $\hat{F}_{\text{acc}}(M)$, is calculated by

$$\hat{F}_{\text{acc}}(M) = \frac{(1 - \epsilon)L_{\text{bol}}(M)}{\epsilon c^2} \int_{t(z=3)}^{t_0} \frac{d\Phi(L, t)}{dL} \frac{dL}{dM} M dt. \quad (27)$$

With the inequality (25), we can estimate the upper limit of the cumulative mass density of SMBHs at $z = 3$ ($F_{z=3}(M)$), $F_{\text{local}}(M) - F_{\text{acc}}(M) - \hat{F}_{\text{acc}}(M)$.

3.2. Tentative SMBHs in LBGs

Here, we conjecture that tentative SMBHs exist in LBGs. In the local universe, the BH mass is not correlated with the disk mass, but with the bulge mass of the galaxy. We note that we tentatively treat the SMBHs in LBGs, which has no clear bulge and disk components (or no clear distinction between these components). Therefore, we simply assume that the mass of SMBHs is related to the stellar mass of LBGs as $M = \xi M_*$, (see Steidel et al. 2002). We treat the mass ratio, ξ , as a free parameter. To compare the proportional relation for the normal galaxies at $z = 0$, $M/M_{\text{bulge}} \sim 0.001 - 0.006$, we examine three cases, $\xi = 0.002, 0.005$, and 0.01 . For the LBGs which have a typical stellar mass ($\sim 10^{10} M_\odot$), the mass of corresponding tentative SMBHs is $10^7 - 10^8 M_\odot$. The cumulative mass density of this tentative SMBHs in LBGs at $z = 3$, $F_{\text{LBG}}(M)$, is given by

$$F_{\text{LBG}}(M) = \int_M^\infty \eta_{\text{LBG}}(M') \frac{d\phi(M_V)}{dM_V} \frac{dM_V}{dM_*} \frac{1}{\xi} dM', \quad (28)$$

where $d\phi(M_V)/dM_V$ is the LFs of LBGs at $z = 3$ in the rest-frame optical (V-band) luminosity and $\eta_{\text{LBG}}(M)$ is the probability that one LBG possesses a SMBH of mass M . The LFs of LBGs are typical Schechter-type functions and we use the one given by Shapley et al. (2001). Shapley et al. also analyze about 100 LBGs with a population synthesis model, assuming a constant star formation rate. They show the relation between the stellar mass of LBGs and apparent luminosity in V-band (see their Fig.13), which is approximately expressed as

$$M_V = -\frac{6}{5} \left(\log \left(\frac{M_*}{M_\odot} h^2 \right) + 8.0 \right) + 5.0 \log(h). \quad (29)$$

Here, we define the mass average of $\eta_{\text{LBG}}(M)$ as

$$\bar{\eta}_{\text{LBG}}(M) \equiv \frac{F_{\text{LBG}}(M)}{\int_M^\infty \frac{d\phi(M_V)}{dM_V} \frac{dM_V}{dM_*} \frac{1}{\xi} dM} \equiv \frac{F_{\text{LBG}}(M)}{\hat{F}_{\text{LBG}}(M)}. \quad (30)$$

We reliably set $F_{z=3}(M) \geq F_{\text{LBG}}(M)$, since there may be other populations of galaxies at $z \sim 3$ besides LBGs and these population may also possess the SMBHs. Very recently, Franx et al.(2003) found the significant population of red galaxies in $z > 2$, which are chosen by their colors, $J_s - K_s > 2.3$. They point that the number density of these red galaxies at $z \sim 3$ can be a half of that of LBGs. Furthermore, van Dokkum et al. (2003) argue that several among these high- z red galaxies display the AGN-like rest-frame UV spectra. More spectroscopy with more samples will make the AGN fraction of these high- z red galaxies clear. With eq.(25), we can calculate the upper limit of $\bar{\eta}_{\text{LBG}}(M)$,

$$\bar{\eta}_{\text{LBG}}(M) \leq \frac{F_{\text{local}}(M) - F_{\text{acc}}(M) - \hat{F}_{\text{acc}}(M)}{\hat{F}_{\text{LBG}}(M)} \equiv \bar{\eta}_{\text{LBG,max}}(M). \quad (31)$$

We again stress that this is a rather loose upper limit due to some reasons. We simply subtract the accreted mass density during $0 < z < 3$ from the mass density of SMBHs found in the center of local galaxies and compare it with that of tentative SMBHs in LBGs. If we can know the merger rates or direct formation rates at each redshift, we can subtract $F_{\text{merge+form}}(M)$ and obtain a more strict upper limit. To investigate the redshift distributions of SMBH-SMBH merger rates and direct formation rates of SMBHs, the gravitational wave (GW)) will be a powerful tool. Today, several powerful GW detectors are under preparation, for example, *Laser Interferometer Satellite Antenna* (LISA) in the low frequency band ($10^{-4} - 10^{-1}$ Hz) and *Laser Interferometer Gravitational-Wave Observatory* (LIGO) in the high frequency band ($10^1 - 10^3$ Hz). Since both the mergers between SMBHs and the gravitational collapse of SMS will emit the low frequency GW (see Haehnelt 1994 for mergers, Saijo et al. 2002 for collapse of SMS), these events are main targets of LISA. Furthermore, these any cosmological events at $z < 10$ should be reliably detected with high signal-to-noise ratio with LISA. Currently LISA is planned to start observations at the earliest in 2011.

3.3. Upper Limit of the Probability that LBGs have SMBHs

3.3.1. Case (i): Non-negligible Contribution of Mergers and/or Direct Formation of SMBHs

We calculate $\bar{\eta}_{\text{LBG,max}}(M)$, the upper limit of the mass averaged probability that one LBG possesses a SMBH more massive than M , by eq.(31). Of course, $\bar{\eta}_{\text{LBG,max}}(M)$ depends on mass conversion efficiency, ϵ , and Eddington ratio, f_{Edd} . First, we assume that these parameters are constants. As discussed in the previous section, in this case, the contribution of mergers and/or direct formation of SMBHs is not negligible and become significant in the low-mass end. Incidentally, many theoretical models for QSO LFs at high redshifts based on the hierarchical structure formation adopted the constant parameters (e.g. Hosokawa 2002, Wyithe & Loeb 2002). Of course, even if we can not neglect the mergers and/or direct formation, the parameters do not have to be constant. Here, we calculate $\bar{\eta}_{\text{LBG,max}}(M)$ with constant parameters as one possible case with non-negligible mergers and/or direct formation of SMBHs. As we have shown in the previous section, we cannot choose these parameters freely since the inequality (9) must be satisfied. We calculate $\bar{\eta}_{\text{LBG,max}}(M)$ with the parameter sets, $(\epsilon, f_{\text{LBG}})$, indicated by points 1 \sim 3 in Fig.2. Below, we choose the mass range of tentative SMBHs to be $10^7 M_{\odot} < M < 10^{8.5} M_{\odot}$ to calculate $\bar{\eta}_{\text{LBG,max}}(M)$. For $M < 10^7 M_{\odot}$, the luminosity range of LBGs corresponds to the faint end of LFs at $z \sim 3$, thus no sufficient data is available. The fitting formulae for LFs of Shapley et al. (2001) are valid only for $M_V - 5 \log(h) < -20.5$. Whereas, the HXLFs of AGNs given by UAOM03

are valid to the sufficiently faint end, $\log L_x = 41.5$, which corresponds to $M \sim 10^5 M_\odot$ if we assume $f_{\text{Edd}} = 1.0$. For $M > 10^{8.5} M_\odot$, conversely, it is difficult to satisfy the inequality eq.(9).

We represent the calculated $\bar{\eta}_{\text{LBG},\text{max}}(M)$ for $\xi = 0.002, 0.005$ and 0.01 in Fig.3 with SMBH mass function (a). In these figures dashed lines represent the mass range where the luminosity of the corresponding LBG is too faint, $M_V - 5 \log(h) > -20.5$. We can see that $\bar{\eta}_{\text{LBG},\text{max}}(M) \sim 1$ is possible for $\xi = 0.002$ with large energy conversion efficiency ($\epsilon \sim 0.3$) (points 2 and 3 in Fig.2) ; that is, almost all LBGs can possess the SMBHs. This value, $\xi = 0.002$, is the same as the ratio between the bulge mass and the SMBH mass found with local galaxies. However, for $\xi = 0.01$ only a part of LBGs can possess the SMBHs ; the upper limits are about 30%. If we use mass function (b), calculated $\bar{\eta}_{\text{LBG},\text{max}}(M)$ becomes larger only by factor 1.5 than that in Fig.3 with similar parameter sets. We can explain why $\bar{\eta}_{\text{LBG},\text{max}}(M)$ increases steeply as M decreases. As mentioned in the previous section, the contribution of mergers and/or direct formation to grow SMBHs increases as M decreases with the constant parameters of mass accretion. We do not include mergers and/or direct formation of SMBHs ($F_{\text{merge+form}}(M)$) in eq.(31) since the event rates of these processes are highly unknown, then the constraint on $\bar{\eta}_{\text{LBG}}(M)$ becomes loose as M decrease. Therefore, if we can know the redshift distribution of merger rates and/or direct formation rates of SMBHs, we can make the upper limit more strict. Especially, if the mergers and/or direct formation occur mainly at $z < 3$, the constraint will be as strict as that with point 1 in Fig.2. In Fig.3, we also plot $F_{\text{local}}(M)/\hat{F}_{\text{LBG}}(M)$ for comparison. This is the upper limit for which the population of tentative SMBHs do not exceed the local population of SMBHs.

Fig.4 represents the mass dependence of $F_{\text{local}}(M)$, $\hat{F}_{\text{LBG}}(M)$ and $F_{\text{local}}(M) - F_{\text{acc}}(M) - \hat{F}_{\text{acc}}(M)$ respectively. These are all cumulative mass densities; $F_{\text{local}}(M)$ is the observational estimate at $z = 0$, $\hat{F}_{\text{LBG}}(M)$ is one assuming that all LBGs have SMBHs with mass ratio, ξ , and $F_{\text{local}}(M) - F_{\text{acc}}(M) - \hat{F}_{\text{acc}}(M)$ is the upper limit at $z = 3$. In this figure, we use the mass function (a) and adopt the parameter set displayed with the points 1-3 in Fig.2. It is clear that if all LBGs have SMBHs with large mass ratio as $\xi = 0.01$, the resultant mass function exceed even the local estimate ($F_{\text{local}}(M)$). The ratio between $\hat{F}_{\text{LBG}}(M)$ and $F_{\text{local}}(M) - F_{\text{acc}}(M) - \hat{F}_{\text{acc}}(M)$ is our calculating $\bar{\eta}_{\text{LBG},\text{max}}(M)$.

We must note that we can only set the *upper* limit of the probability, in principle, that LBGs have SMBHs. Then, we can not even rule out that any normal LBGs (not AGNs) do not have SMBHs. Here, we conclude as follows. If we assume the mass ratio, $\xi (\equiv M_{\text{BH}}/M_*)$, of tentative SMBHs in LBGs is comparable to be observed $M_{\text{BH}}/M_{\text{bulge}}$ of normal galaxies at $z = 0$, we cannot completely rule out that the large part of LBGs possess inactive SMBHs. However, some conditions are needed in this case. That is, large energy conversion efficiency

($\epsilon \sim 0.3$) and significant mergers and/or direct formation of SMBHs at high redshifts ($z > 3$). Whereas, if ξ is larger than the local value of $M_{\text{BH}}/M_{\text{bulge}}$, the fraction of LBGs which can possess SMBHs decrease to less than several tens of %.

3.3.2. Case (ii): Dominant Contribution by Mass Accretion to Grow SMBHs

Some recent works suggest that the mass accretion may be the dominant process to grow SMBHs (Yu & Lu 2004, Marconi et al. 2004). Here, we consider the special case, where the mergers and direct formation of SMBHs through gravitational collapse of the SMS are not important for $M \geq 10^7 M_{\odot}$. Concretely, $G_{\text{merge+form}}(M, t_0)$ can be neglected and the equality is satisfied in eq.(9),

$$G_{\text{local}}(M, t_0) = G_{\text{acc}}(M, t_0). \quad (32)$$

In this case, we differentiate eq.(32) by M and we can represent the energy conversion efficiency as the function of M ,

$$\frac{\epsilon}{1 - \epsilon}(M) = \frac{\frac{L_{\text{bol}}(M)}{c^2} \int_0^{t_0} \frac{d\Phi}{dL}(L(M), t) \frac{dL}{dM} dt}{\int_M^{\infty} n(M', t_0) dM'}, \quad (33)$$

(YT02). Inversely, if one calculate $G_{\text{local}}(M, t_0)$ and $G_{\text{acc}}(M, t_0)$ using $\epsilon(M)$ with eq.(33), eq.(32) is naturally satisfied in all mass range. The functional form of $\epsilon(M)$ is the same as that in Fig.4. of YT02. Here, we calculate $\bar{\eta}_{\text{LBG, max}}(M)$ with the mass dependent energy conversion efficiency, $\epsilon(M)$ given by eq.(33). We note that when mergers and direct formation of SMBHs are negligible, the equality is satisfied in eq.(25). However, the inequality $F_{z=3}(M) \geq F_{\text{LBG}}(M)$ still remains due to possible other populations of hosts of SMBHs besides LBGs. Therefore, $\bar{\eta}_{\text{LBG, max}}(M)$ calculated with $\epsilon(M)$ still gives the upper limit of $\bar{\eta}_{\text{LBG}}(M)$ though it is more stringent than the case with constant parameters.

Fig.5 represents the results. As expected, the calculated $\bar{\eta}_{\text{LBG, max}}(M)$ is more strict than the results with constant parameters (Fig.3). Even if the value of ξ is comparable to $M_{\text{BH}}/M_{\text{bulge}}$ (0.001-0.006) of local galaxies, only a part of LBGs are allowed to have SMBHs (less than 10-30%). Especially, if ξ is larger than $M_{\text{BH}}/M_{\text{bulge}}$, LBGs which possess SMBHs become very rare (less than 10 %). If this is the case, 3% of LBGs which show the AGN activity may be the special LBGs where SMBHs are born within the galaxies. In Fig.5, we also plot $F_{\text{local}}(M)/\hat{F}_{\text{LBG}}(M)$ for reference. These lines are same as Fig.3. In this case, the difference between calculated $\bar{\eta}_{\text{LBG, max}}(M)$ and $F_{\text{local}}(M)/\hat{F}_{\text{LBG}}(M)$ is large. In Fig.4, we can see that $F_{\text{local}}(M) - F_{\text{acc}}(M) - \hat{F}_{\text{acc}}(M)$ is smaller than $F_{\text{local}}(M)$ by about one order of magnitude in $10^7 M_{\odot} < M < 10^{8.5} M_{\odot}$.

3.3.3. Necessary Condition to Neglect Mergers and/or Direct Formation

As we show above, how much mergers and direct formation of SMBHs occur until $z = 0$ is important to estimate the mass function of SMBHs at high redshifts ($z \sim 3$ in this paper). As mentioned above, we can observe these events by the gravitational radiation with LISA in the future. Here, we crudely estimate the necessary condition to drop the merger term in eq.(5). In eq.(2) $\Omega(M_1, M_2)$ is the number of merger events among the SMBHs of mass M_1 and M_2 per unit volume per unit cosmological time. We can express this $\Omega(M_1, M_2)$ as,

$$\Omega(M_1, M_2) = \frac{\Delta N}{\Delta M_1 \Delta M_2 \Delta t \Delta V}. \quad (34)$$

With this expression, the contribution of mergers to $G_{\text{merge+form}}(M, t_0)$ may be written as

$$G_{\text{merge}}(M, t_0) \equiv \int_M^\infty \int_0^{t_0} f(M, M') \gamma_{\text{merge}}(M', t) dM' dt \sim \frac{M' \Delta N}{\Delta \ln M \Delta \ln M' \Delta z \Delta V} \cdot \Delta z \quad (35)$$

Here, we transform the time interval, Δt to the redshift interval Δz . If $G_{\text{merge}}(M, t_0)$ is less than $G_{\text{local}}(M, t_0)$ and $G_{\text{acc}}(M, t_0)$, say, at $M \sim 10^7 M_\odot$, we have

$$\frac{\Delta N}{\Delta \ln M \Delta \ln M' \Delta z \Delta V} \cdot \Delta z \leq 10^{-2} (\text{Mpc}^{-3}), \quad (36)$$

where we use $G_{\text{local}} \sim G_{\text{acc}} \sim 10^5 M_\odot \cdot \text{Mpc}^{-3}$ at $M \sim 10^7 M_\odot$ (see Fig.1) and we set $M' \sim 10^7 M_\odot$. Of course, other mergers than $M \sim M' \sim 10^7 M_\odot$ contribute to $G_{\text{merge}}(10^7 M_\odot, t_0)$, but we estimate loose upper limit of the event rate of mergers where $M \sim M' \sim 10^7 M_\odot$ to drop the merger term in eq.(5). We can convert eq.(36) to the number of the events which we can observe per unit time from the earth,

$$\frac{dN}{d \ln M d \ln M' dz dt} \cdot dz = \frac{\Delta N}{\Delta \ln M \Delta \ln M' \Delta z \Delta V} \cdot \Delta z \times 4\pi c(1+z) \cdot d_A^2(z), \quad (37)$$

where $d_A(z)$ is the angular diameter distance. The time interval dt on the left-hand side is related to the volume interval ΔV on the right-hand side as $\Delta V = 4\pi c(1+z)d_A^2 dt$. If we set $z = 3$, the angular diameter distance is $d_A(z = 3) \sim 1 \text{Gpc}$ in the ΛCDM universe (the redshift dependence of d_A is not large), then we have

$$\frac{dN}{d \ln M d \ln M' dz dt} \cdot dz \leq 0.6 \quad (\text{times} \cdot \text{yr}^{-1}) \quad (38)$$

with eq.(36). This is a necessary condition for event rate of mergers of $M \sim M' \sim 10^7 M_\odot$ to drop $G_{\text{merge}}(10^7 M_\odot, t_0)$ in eq.(5). That is, if we observe one or more events of the SMBH-SMBH merger between $10^7 M_\odot$ SMBHs per one year, the effect of mergers to grow SMBHs is at least not negligible, compared with the mass accretion. To neglect the mergers in terms of the continuity equation of SMBHs, no detection of events for several years is at least needed. We can determine the contribution of mergers observationally in the future.

4. Discussions

4.1. Scatter in the $M - \sigma$ Relation

In §2.2, we have searched the parameter sets, $(\epsilon, f_{\text{Edd}})$ which satisfy inequality (9) with the HXLFs of AGNs (Fig.2). In fig.2, we can find the allowed region of the parameter space where ineq.(9) is satisfied only in $M \leq 10^{8.5} M_{\odot}$. However, if we demand sub-Eddington case, $f_{\text{Edd}} \leq 1.0$, it is still difficult to satisfy ineq.(9) in the high-mass end ($M \geq 10^9 M_{\odot}$). For example, even if we adopt the most adequate parameter set, $(\epsilon, f_{\text{Edd}}) = (0.3, 1.0)$ to suppress $G_{\text{acc}}(M, t_0)$, ineq.(9) is satisfied only in $M \leq 10^9 M_{\odot}$ (see Fig.1). One possible resolution of this discrepancy is the scatter included in the mass function of the SMBHs. That is, the scatter in the $M - \sigma$ relation. Fortunately, Yu & Lu (2004) study the effect of the scatter on the mass function. They assume the Gaussian scatter for the distribution of $\log M_{\text{BH}}$ at a given σ (see their eq.(43)), and find that only the high-mass end of the mass function ($> 3 \times 10^8 M_{\odot}$) is sensitive to the scatter. Furthermore, the scatter (they test the 3 cases for the intrinsic standard deviation, $\Delta_{\log M_{\text{BH}}} = 0, 0.27$ and 0.4) *increase* the high-mass end (see their Fig.1). Especially, they show that the effect of the scatter is significant in $M \geq 10^9 M_{\odot}$. This is an important result. The discrepancy discussed above will be resolved with the accurate decision of the intrinsic scatter in the $M - \sigma$ relation. Incidentally, we note that the fitting function of the mass function by AR02 used in this paper becomes worse in the high-mass end ($M > 10^{8.5} M_{\odot}$) (see their Fig.8 and 9).

4.2. Relaxing Assumptions

In this paper, we use some assumptions for simplicity. Here, we discuss the effects of relaxing these assumptions. Some of these assumptions are listed as following,

- (i) extrapolation of HXLFs of AGNs to $z > 3$,
- (ii) no redshift evolution of ϵ and f_{Edd} ,
- (iii) mass conservation during mergers.

Throughout this paper, we extrapolate the observed HXLFs which are valid only at $z < 3$ to the higher redshifts (assumption (i)). However, we note that the calculations in this paper depend only on the HXLFs just before $z = 3$. To confirm this, we perform the same calculations with the tentative HXLFs, $d\Phi(L_X, z)/d\log L_X = 0$ for $z > 3.5$. First, the critical value of ϵ for a given f_{Edd} to satisfy ineq.(9) becomes less than 15% smaller than that

of Fig.2. Next, $\bar{\eta}_{\text{LBG,max}}(M)$ becomes less than 5% smaller than that of Fig.3. Therefore, we conclude that the results in this paper are robust unless the HXLFs change suddenly from the extrapolation of the observed ones at $z < 3$ just before $z = 3$.

Since the redshift evolution of the parameters, ϵ and f_{Edd} is uncertain, we adopt the assumption (ii). Due to this assumption, we avoid to introduce extra parameters to represent the redshift evolution. For the energy conversion efficiency, ϵ , we have no idea on the redshift evolution. Many theoretical works adopt ϵ with no redshift evolution (e.g. Kauffmann & Haehnelt 2000, Wyithe & Loeb 2002, Volonteri, Haardt & Madau 2003). For Eddington ratio, f_{Edd} , several theoretical works argue that f_{Edd} decreases as redshift decreases and that this is one possible origin of the rapid decline in population of bright QSOs from $z \sim 2.5$ to $z = 0$ (e.g. Kauffmann & Haehnelt 2000, Haiman & Menou 2000). Therefore, we test the following simple decline of f_{Edd} to relax assumption (ii),

$$f_{\text{Edd}}(z) = \begin{cases} 1.0, & \text{for } z > 2.5 \\ \frac{1.0 - f_{\text{Edd},0}}{2.5}z + f_{\text{Edd},0}, & \text{for } z \leq 2.5 \end{cases} \quad (39)$$

We calculate $G_{\text{acc}}(M, t_0)$ for $f_{\text{Edd},0} = 0.5, 0.1, 0.05$ and 0.01 . However, the calculated $G_{\text{acc}}(M, t_0)$ with $f_{\text{Edd}}(z)$ is different from that with constant f_{Edd} only by less than 10%. In this case, the best-fit values of constant f_{Edd} are $f_{\text{Edd}} = 0.8, 0.64, 0.62$ (corresponding to $f_{\text{Edd}}(z = 1.5)$) and 0.5 ($f_{\text{Edd}}(z = 1.25)$) for $f_{\text{Edd},0} = 0.5, 0.1, 0.05$ and 0.01 respectively.

During the merger of the SMBHs, the strong GW is emitted. Actually, the total mass of the SMBH binary can generally decrease due to the GW emission during the merger despite the assumption (iii). For the most efficient radiative merging, the total entropy (or the area) of SMBHs is conserved. In this case, the resultant mass of the merger between SMBHs of M_1 and M_2 is $M = \sqrt{M_1^2 + M_2^2}$. That is, about 30% of $M_1 + M_2$ can be maximally radiated as the GW. In this paper, we constrain the upper bound of the population of SMBHs at $z = 3$ based on the assumption that the mass density of SMBHs never decreases. If significant mergers decrease the mass of SMBHs, we may not rule out that there are already much SMBHs at $z = 3$ than that at $z = 0$. YT02 consider the growth history of SMBHs for the maximally efficient radiative merging using the continuity equation. Here, we only note that the formulation in this paper can be applied for this case by replacing the variable, M with the entropy (or area) of the SMBHs, though there is the uncertainty on the spin parameter, a .

4.3. Other Possible Constraint

The current concrete observational information on the “active” LBGs is only the number fraction to the all LBGs, 3% (Steidel et al. 2002, Nandra et al. 2002). However, we can possibly use this fraction, 3% as the lower limit of $\bar{\eta}_{\text{LBG,max}}(M)$. Though 3% is much smaller than the calculated values of $\bar{\eta}_{\text{LBG,max}}(M)$ in many cases, if this fraction increases in the future, we can get the more strict lower limit. For the dominant mass accretion case (§3.3.2 or Fig.5), especially, calculated $\bar{\eta}_{\text{LBG,max}}(M)$ is comparatively small, then the lower limit will be valid. If we can put the meaningful lower limit as noted above, we can constrain the lower bound of Eddington ratio, f_{Edd} and the upper bound of the mass ratio, ξ . In all cases, calculated $\bar{\eta}_{\text{LBG,max}}(M)$ decreases to the high-mass end. However, we must consider the scatter of the $M - \sigma$ relation in the high-mass end to apply the lower limit (§4.1).

5. Conclusions

In this paper, we study demographics of SMBHs in the local and high- z universe and crudely constrain the possibility that LBGs have SMBHs for an application. To constrain the possible population of SMBHs within the LBGs, we estimate the upper limit of mass density of SMBHs at $z \sim 3$. In other words, if there are too many and too massive SMBHs already at $z \sim 3$, the resultant mass density of SMBHs at $z \sim 0$ exceed the local mass density of SMBHs. We use the continuity equation of SMBHs, eq.(1) and transform it to the useful integrated form. We consider the three physical processes to grow the SMBHs; mass accretion, mergers and direct formation of SMBHs. Of these processes to grow SMBHs, we can use the HXLFs of AGNs to estimate the cosmological accretion history into the SMBHs.

First, we investigate the demographics of SMBHs in the local universe. YT02 argue that inequality (9) is not satisfied with the accreted mass at $z > 0$ estimated with OLFs of QSOs and with the local mass function of SMBHs with typical parameter set, $(\epsilon, f_{\text{Edd}}) = (0.1, 1.0)$. We use the HXLFs of AGNs including optically absorbed AGNs, then the accreted mass with HXLFs should have become larger than that with OLFs. However, we show that the discrepancy is resolved using HXLFs with the same parameters and constant L_X/L_{bol} . The origin of the discrepancy is probably the assumption of the constant L_B/L_{bol} . The comparison between the redshift evolution of OLFs and that of HXLFs suggests that $L_B \propto L_X^{1.4}$. Therefore, if the assumption of the constant L_X/L_{bol} is reasonable, the ratio of B-band luminosity to the bolometric luminosity has the dependence as $L_B/L_{\text{bol}} \propto L_{\text{bol}}^{-0.4} \propto M^{-0.4}$. This makes the redshift evolution of OLFs as slow as that of HXLFs as z increases, and calculated $G_{\text{acc}}(M, t_0)$ with OLFs becomes small. Furthermore, we constrain the energy conversion efficiency, ϵ and Eddington rate, f_{Edd} to satisfy the inequality (9) upto the high

mass end ($M < 10^8 M_\odot, 10^{8.5} M_\odot$). If we demand $f_{\text{Edd}} \leq 1$, the allowed region of $(\epsilon, f_{\text{Edd}})$ exist only if we use HXLFs ($\epsilon \sim 0.1 - 0.3$ and $f_{\text{Edd}} \sim 0.3 - 1.0$). Totally, we conclude on the demographics of SMBHs in the local universe as follows.

- The cosmological accretion history based on HXLFs is more plausible than that based on OLFs. This is because we can satisfy ineq.(9) with typical parameters (especially $f_{\text{Edd}} \leq 1$). The reason why calculated $G_{\text{acc}}(M, t_0)$ with HXLFs is smaller than that with OLFs is that we assume constant luminosity ratio, C_B and C_X for both bands.

With these constant parameter sets, $G_{\text{local}}(M, t_0) - G_{\text{acc}}(M, t_0)$ becomes large at the low mass end. This means that the contribution of mergers and/or direct formation of SMBHs is important in the low mass end, if the physical parameters are actually constant as we assumed. We adopt the constant parameters to estimate the accreted mass to SMBHs during $0 < z < 3$ for the case that there is non-negligible mergers and/or direct formation in the latter-half of this paper.

Next, we derive the inequality (25) between the population of SMBHs at $z = 0$ and $z = 3$ and the accreted mass into SMBHs during $0 < z < 3$ from eq.(1) again. This gives the upper limit of the cumulative mass density of SMBHs at any high redshifts. For an application, we consider the tentative SMBHs within the LBGs. We assume that the mass of SMBHs, M is proportional to the stellar mass of LBGs, M_* with the parameter $\xi \equiv M/M_*$. We define the mass averaged probability that one LBG harbors a SMBH more massive than M , $\bar{\eta}_{\text{LBG}}(M)$, and calculate the upper limit of $\bar{\eta}_{\text{LBG}}(M)$, $\bar{\eta}_{\text{LBG,max}}(M)$. We simply estimate the upper limit of cumulative mass density of SMBHs at $z \sim 3$ considering only mass accretion during $0 < z < 3$. For the event rate of mergers and/or direct formation of SMBHs, we consider two simple limit; (i) there are non-negligible contribution of mergers and/or direct formation (ii) the mass accretion is the dominant process to grow the SMBHs. The constraint on the population of the tentative SMBHs in LBGs are summarized as follows.

- In case (i), we adopt the constant parameters for one possible case as mentioned above. If the assumed mass ratio, ξ is comparable to M/M_{bulge} observed in the local galaxies ($\xi = 0.002$), we cannot rule out that almost all LBGs can have SMBHs ($\bar{\eta}_{\text{LBG,max}}(M) \sim 1.0$). However, if this is the case, the large energy conversion efficiency, $\epsilon \sim 0.3$ and significant mergers and/or direct formation of SMBHs at $z > 3$ will be needed. If ξ is larger than the local value ($\xi > 0.005$), only a part of LBGs can have SMBHs ($\bar{\eta}_{\text{LBG,max}}(M) \sim \text{several ten \%}$).
- In case (ii), we can make the upper limit more strict. In this case, we can express the energy conversion efficiency as the function of the SMBH mass, $\epsilon(M)$. With this $\epsilon(M)$,

calculated $\bar{\eta}_{\text{LBG,max}}(M)$ is only ten % even if ξ is small. Especially, if ξ is larger than the M/M_{bulge} of local galaxies, $\bar{\eta}_{\text{LBG,max}}(M)$ can become less than 10%.

We roughly estimate the necessary condition to neglect the merger term in eq.(5). For example, for the major merger between SMBHs of $10^7 M_{\odot}$, the loose upper limit is about 1 event per one year. If we observe these merger events one or more times per year, we can not neglect the SMBH-SMBH mergers. Totally, the possibility that nearly all LBGs have SMBHs with large mass ratio, $\xi > 0.005$, is reliably ruled out. Even if AGN-like LBGs have SMBHs with large mass ratio as Vestergaard (2003) and Vestergaard (2004) suggested, remaining normal LBGs will not have inactive SMBHs with the same mass ratio.

Though we consider the tentative SMBHs in LBGs with mass ratio, $\xi \equiv M/M_*$, this proportional relation is only a trial one. Papovich et al.(2001) note that the stellar mass estimated with the rest-frame UV-to-optical spectrum is only the mass of young stars. That is, additional population of old stars may exist. Papovich et al. estimate that the upper limit of the mass of old stars is 3-8 times as large as that of young stars. Since the SMBH mass is not correlated with the younger disk component but older bulge component in the nearby galaxies, it may be better to relate the tentative SMBH mass with the mass of old stars if they exist. However, our approach will be still valid, since we simply try another values of ξ . Of course, even if SMBHs actually exist in the high- z galaxies, the proportional relation between M and M_{bulge} itself is never guaranteed. Fortunately, it is known that M/M_{bulge} of nearby AGNs is the same as that of local galaxies (Wandel 2002). It will be the first step to investigate the relation between the stellar mass and SMBH mass of LBGs which show the AGN activity.

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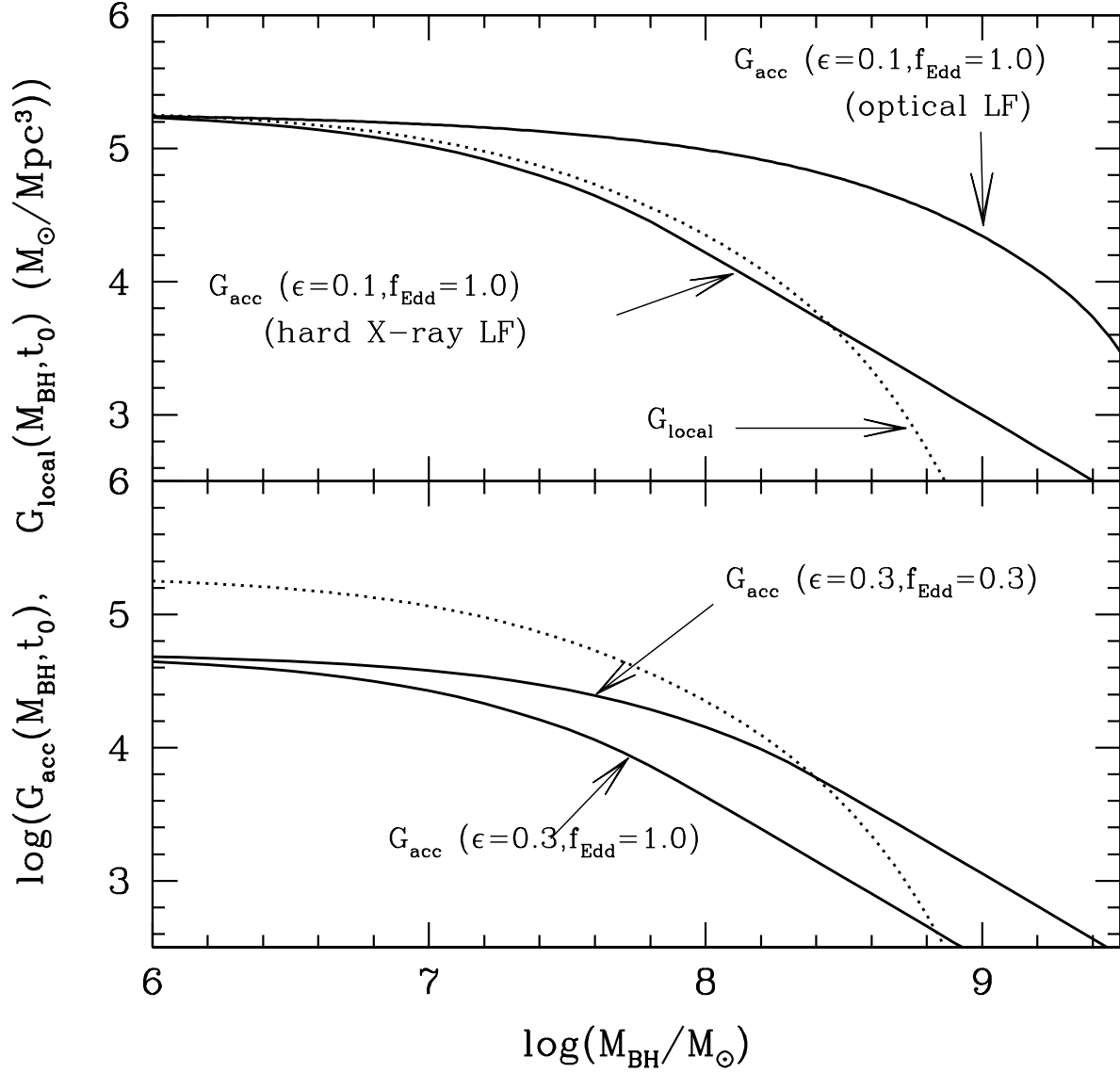


Fig. 1.— Calculated $G_{\text{acc}}(M, t_0)$ and $G_{\text{local}}(M, t_0)$ using the local mass function of SMBHs (a) (see text in detail). *Upper panel:* $G_{\text{acc}}(M, t_0)$ with the typical parameter set, $(\epsilon, f_{\text{Edd}}) = (0.1, 1.0)$ using OLFs of QSOs and HXLFs of AGNs respectively, and $G_{\text{local}}(M, t_0)$. *Lower panel:* $G_{\text{acc}}(M, t_0)$ with the parameter sets indicated by points 2 $((\epsilon, f_{\text{Edd}}) = (0.3, 1.0))$, 3 $((0.3, 0.3))$ in Fig.2. $G_{\text{local}}(M, t_0)$ is the same as upper panel.

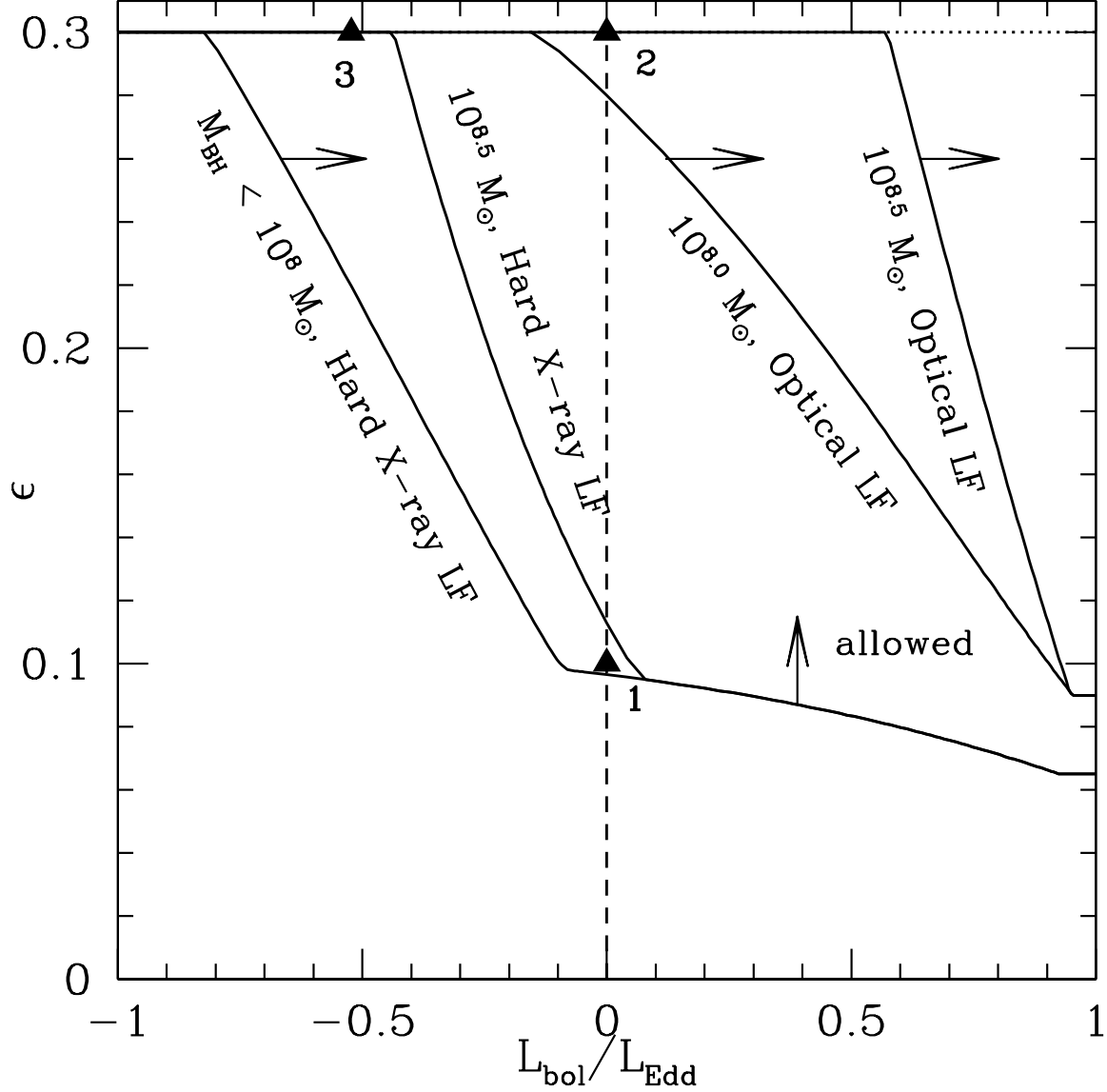


Fig. 2.— The region of the parameter space where the inequality, $G_{\text{local}}(M, t_0) \geq G_{\text{acc}}(M, t_0)$ is satisfied only at $M \leq M_0$. We calculate $G_{\text{local}}(M, t_0)$ using the local mass function of SMBHs (a) (see text in detail). The allowed region for each M_0 is above the labeled lines for $M_0 = 10^{8.0}$ and $10^{8.5} M_\odot$. We set the upper limit of the energy conversion efficiency as $\epsilon = 0.3$. The representative parameter sets are indicated by points 1 ~ 3. We can show the corresponding $G_{\text{acc}}(M, t_0)$ for 1 ~ 3 in Fig.1.

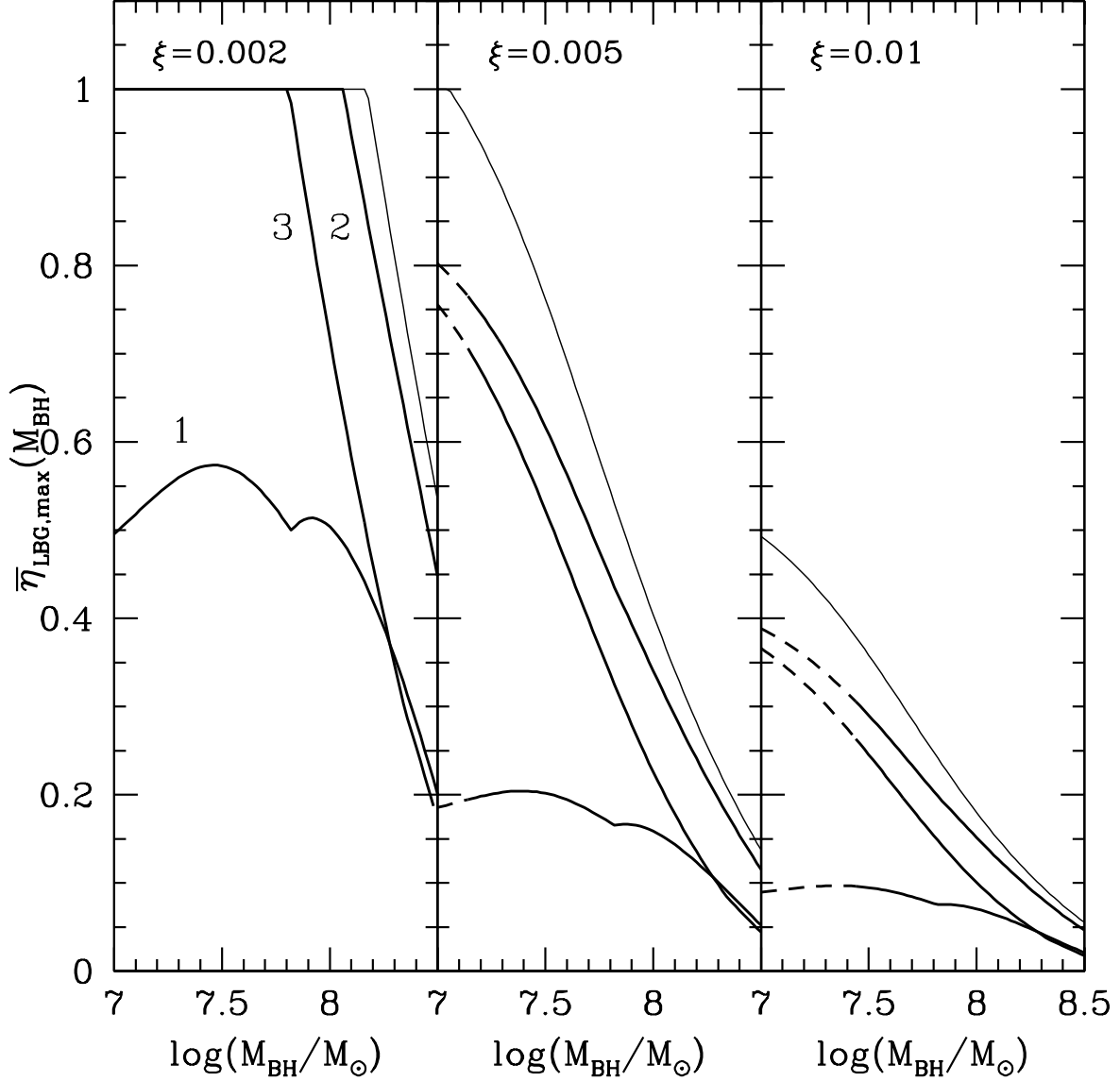


Fig. 3.— Mass averaged probability that one LBG possess a SMBHs more massive than M , $\bar{\eta}_{\text{LBG,max}}(M)$ for $\xi = 0.002, 0.005$ and 0.01 . The lines represent $\bar{\eta}_{\text{LBG,max}}(M)$ calculated with parameter sets indicated in Fig.2 by points 1 \sim 3 (left to right; point 1, 3, 2 in turn). In the figure, dashed lines represents the mass range where the luminosity of the corresponding LBG is too faint : $M_V - 5 \log(h) > -20.5$. The thin solid lines represents $F_{\text{local}}(M)/\hat{F}_{\text{LBG}}(M)$.

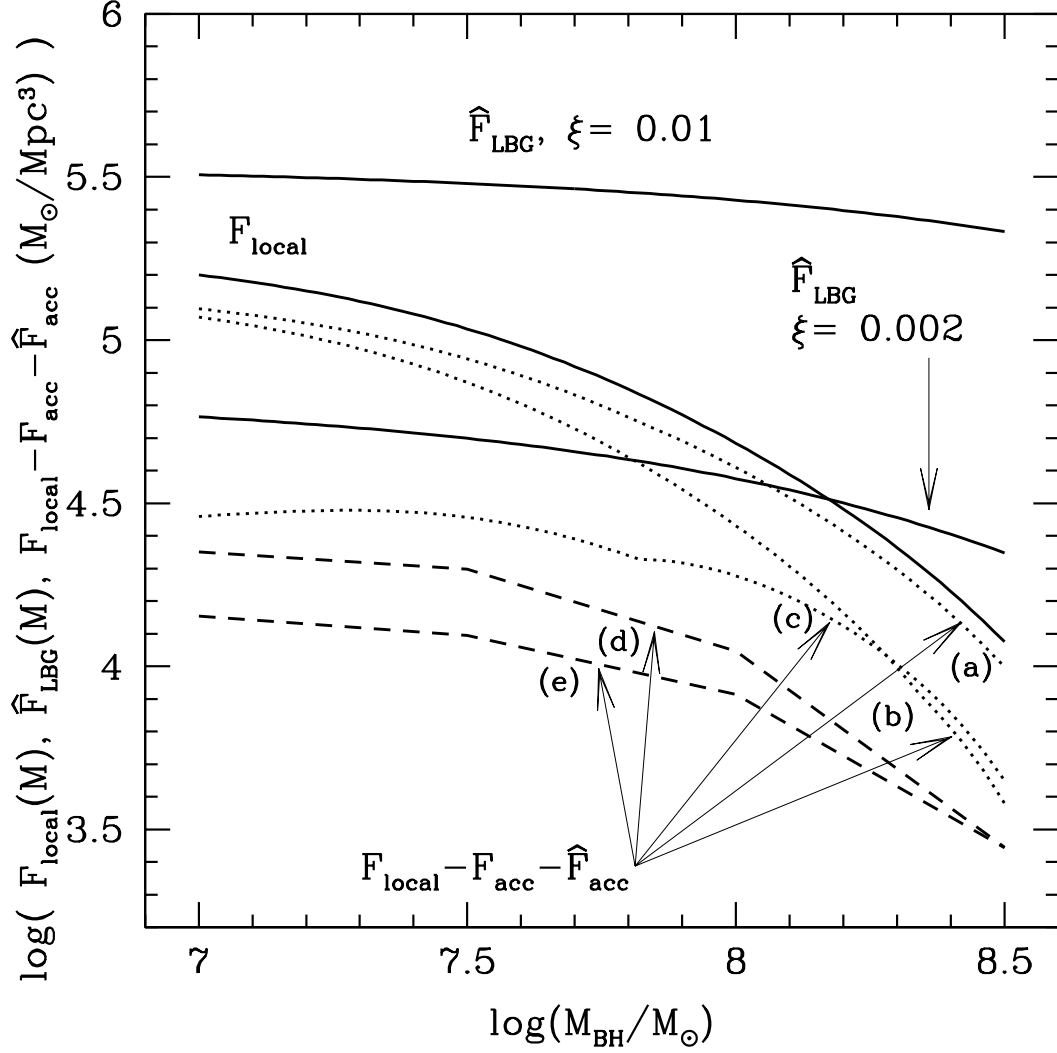


Fig. 4.— The mass dependence of the cumulative mass densities of SMBHs; $F_{\text{local}}(M)$: observational estimate at $z = 0$, $\hat{F}_{\text{LBG}}(M)$: assuming that all LBGs have SMBHs with mass ratio, $\xi = 0.01$ and 0.002 , at $z = 3$, $F_{\text{local}}(M) - F_{\text{acc}}(M) - \hat{F}_{\text{acc}}(M)$: the upper limit at $z = 3$. We represent $F_{\text{local}}(M) - F_{\text{acc}}(M) - \hat{F}_{\text{acc}}(M)$ in five cases; (a): with constant parameters indicated by point 2 in Fig.2 , (b): same as (a) but for point 3, (c): same as (a) but for point 1, (d): with mass dependent energy conversion efficiency (eq.(33)) and $f_{\text{Edd}} = 1.0$, (e): same as (d) but for $f_{\text{Edd}} = 0.3$.

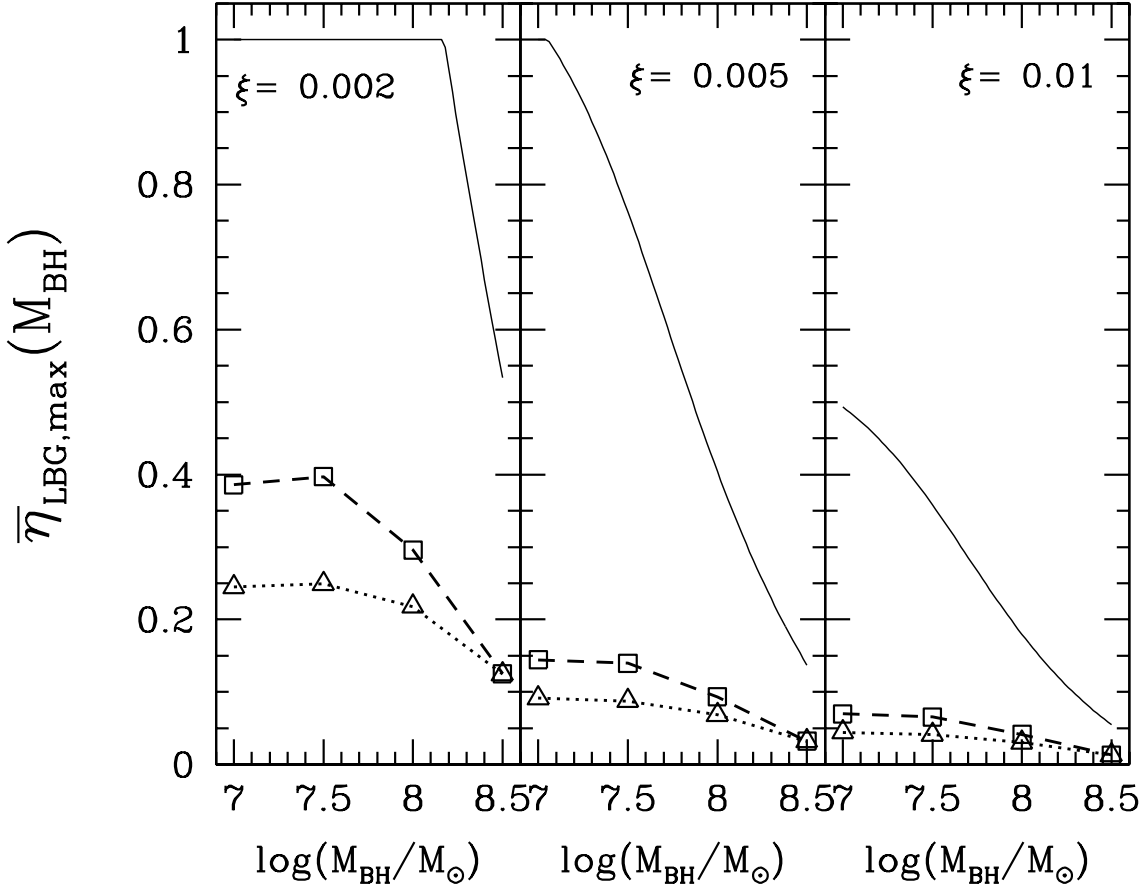


Fig. 5.— Mass averaged probability that one LBG possess a SMBHs more massive than M , $\bar{\eta}_{\text{LBG,max}}(M)$ with the mass dependent energy conversion efficiency given by eq.(33) for $\xi = 0.002, 0.005$ and 0.01 respectively. Dashed lines and dotted lines represents the results with $f_{\text{Edd}} = 1.0$ and 0.3 respectively. We calculate $\bar{\eta}_{\text{LBG,max}}(M)$ only at $M = 10^7, 10^{7.5}, 10^8$ and $10^{8.5}M_{\odot}$ and connect these values with lines. The thin black solid lines represents $F_{\text{local}}(M)/\hat{F}_{\text{LBG}}(M)$.